Finance Research Letters 19 (2016) 228–234

Contents lists available at ScienceDirect

Finance Research Letters

journal homepage: www.elsevier.com/locate/frl

Credit risk findings for commercial real estate loans using the reduced form

Andreas D. Christopoulos[a,*], Joshua G. Barratt[b]

[a] University of Texas at Austin, McCombs School of Business, Austin, TX 78712-1276, United States
[b] Barratt Consulting, Berlin, Germany

ARTICLE INFO

Article history:
Received 17 July 2016
Revised 2 August 2016
Accepted 10 August 2016
Available online 12 August 2016

JEL Classification:
G12
G13

Keywords:
CMBS
Credit risk
Financial crisis
Mortgages
MBS
Default
PD
Expected loss

ABSTRACT

This paper considers probability of default and expected loss profiles of 25,019 mortgages collateralized by commercial real estate properties evaluated using a reduced form model on a daily basis over the period November 2007 through January 2015. Our evaluations provide a compact and valuable set of insights to build intuition on credit risks facing CMBS investors.

1. Introduction

The commercial mortgage backed securities (CMBS) market can be characterized by the complexity of the risks underlying the commercial real estate loan (CREL) collateral that secures bond cashflows. At the core of the endeavor of modelling such risks is the so-called 'default-trigger', the threshold at which under simulated conditions a loan's promised cashflows will be perturbed and reflect pay-off timing and amounts that differ from the originally promised schedule.

The purpose of this paper is to build intuition on the probability of default and expected loss values, determined at the loan and trust levels, using a reduced form approach for a large sample of 25,019 CRELs spanning the period November 2007 through January 2015. As the data in CMBS is non-public, and as the CMBS market does not utilize derivatives pricing technology as discussed in Christopoulos et al. (2014), the findings in this short paper are rare in the academic literature.

The remainder of the paper is organized as follows. In Section 2 we provide discussion of the data. In Section 3 we discuss the credit risk implications while in Section 4 we provide the results. We conclude in Section 5 and in the Appendix we provide a synopsis of key elements of Christopoulos and Jarrow (2016) which this analysis and simulation output is based upon.

* Corresponding author.
E-mail addresses: andreas.christopoulos@mccombs.utexas.edu (A.D. Christopoulos), joshua@barratt.pro (J.G. Barratt).

http://dx.doi.org/10.1016/j.frl.2016.08.004
1544-6123/

Table 1
Summary of CMBX and Underlying Loans: This table summarizes the number of loans by property type across different CMBX series evaluated in this study and the total loan balance (in $ billions).

	CMBX Ser 1	CMBX Ser 2	CMBX Ser 3	CMBX Ser 4	CMBX Ser 5	CMBX Ser 6	CMBX Ser 7	CMBX Ser 8	CMBX Ser 1:8
Industrial	547	529	613	581	447	200	189	249	3355
Lodging	256	401	437	446	283	238	225	230	2516
Multifamily	701	387	378	347	162	251	482	496	3204
Office	834	809	933	974	634	269	192	290	4935
Other	107	158	228	248	205	85	88	99	1218
Retail	1559	1606	1833	1822	1414	544	491	522	9791
Total (#)	4004	3890	4422	4418	3145	1587	1667	1886	25,019
Total ($ billions)	$57.8	$55.1	$71.9	$72.2	$49.9	$29.9	$27.5	$25.1	$389.4

2. Data

The costly non-public data on loans and bonds was donated by an institutional investor with ˜$0.50 trillion assets under management (AUM) for the purposes of this study. The CRELs span origination dates 2001 through 2015. The data vendor is Intex and the frequency of the CREL data is monthly. Loan level characteristics such as property type, location, loan coupon, and size are included as are, importantly, the updated payment statuses of the loans which may be characterized as current, delinquent, or default. With a principal balance of $389 billion, our sample represents approximately 1/3 of the entire CMBS universe outstanding and nearly all of the actively traded CMBS collateral. Table 1 provides a summary of the loans included in our study. Additional non-public data provided by the institutional investor included property x regional indices published by the National Council of Real Estate Investment Fiduciaries (NCREIF). Public data on interest rates was secured from the Federal Reserve's H.15. Selected Interest Rates website[1] and public data on real estate investment trusts (REITs) was provided through the various stock exchanges on which they are listed.

3. Credit risk implications

Christopoulos and Jarrow (2016) capture the likelihood of transition from a delinquent payment state to a default payment state, λ_f, as described in the Appendix in Eq. (5) as a default intensity under the probability measure $\mathbb{Q}$. Default may only be arrived at from a state of delinquency, over the discrete time interval $[t, t+\Delta]$. We implement their reduced form model on a daily basis over the period November 2007 through January 2015 for the purpose of capturing the expected loss profile for all loans in our sample for this study.[2]

We utilize a static loss severity assumption by property type as observed in the Intex data. The loss severities we use, which vary by property type, are: Multifamily (36%); Retail (47%); Office (37%); Hotel (48%); Industrial (38%); and Other (50%).[3] We assume three months to disposition for all loans that transition to default. The static choices are consistent with Driessen and Van Hemert (2012), The Bank for International Settlements (2008), Peaslee and Nirenberg (2001), and other studies.

The prospects for losses under simulation using default intensities should tend to be downward sloping as the remaining time to maturity for loans diminishes, all else being equal. For the default intensity, λ_f, the expected remaining lifetime default rate, $E[Def_{i,t}]$, for the ith loan at any time t with loan maturity date T is given by

$$E[Def_{i,t}] = \int_t^T \lambda_f(t)dt \tag{1}$$

The implication of a downward slope observed in the empirical results (which we discuss below) should follow because

$$\int_0^T \lambda_f(t)dt > \int_{t_1}^T \lambda_f(t)dt > \cdots > \int_{T-\Delta t}^T \lambda_f(t)dt \tag{2}$$

As loans age and maturity draws closer, the number of months simulated at any simulation time declines. Specifically, as the interval $[t, T]$ becomes smaller there are, generally, less and less possibilities for the diffusions of the economy that interact with loan characteristics in the default intensity, λ_f, to generate defaults.

To confirm this perspective we apply the static property-type specific loss severity assumptions, $L_{i,k}$, to each of the i loans of the kth property type at each default time in the simulation such that the expected loss for the ith loan at any time t with loan maturity date T is given by

$$E[Loss_{i,t}] = L_{i,k} \times E[Def_{i,t}] \tag{3}$$

[1] https://www.federalreserve.gov/releases/h15/data.htm
[2] Key aspects of that study are provided in the Appendix to this study.
[3] These compare well with Moody's historical loss severities for CRELs across all vintages of Multifamily (35%); Retail (49%); Office (40%); Hotel (46%); and Industrial (39%) as reported in Banhazl and Halpern (2015) and also correspond to similar values reported by Wells Fargo (see, Frerich and van Heerden, 2015).

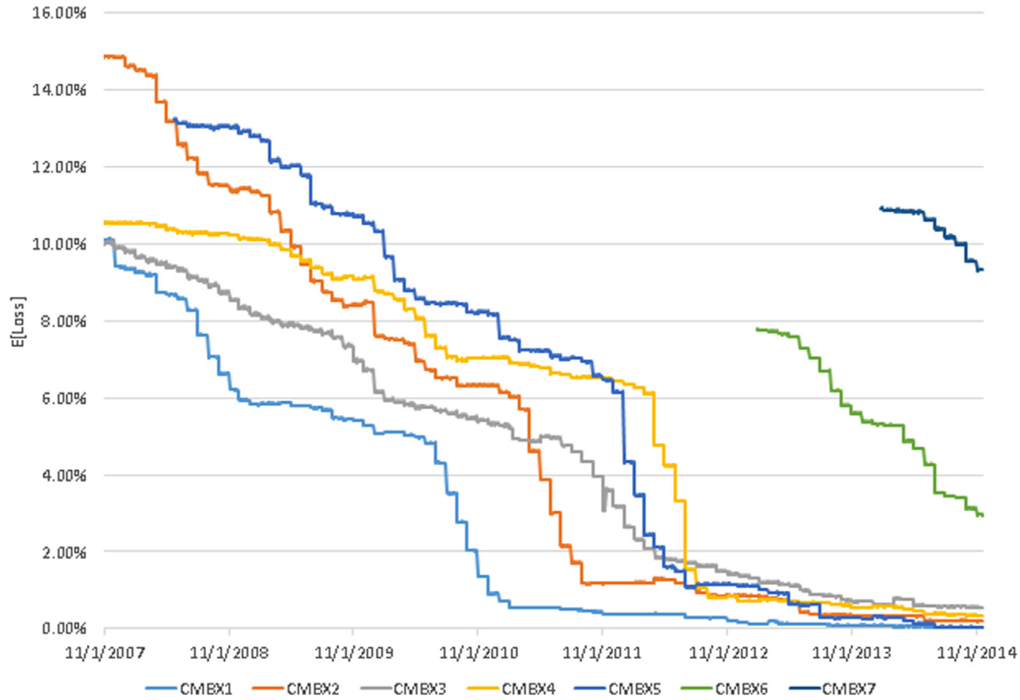

Fig. 1. Expected loss, CMBX Series 1 through 7: This chart shows the simulated aggregated expected loss for CMBX Series 1 through 7.

As there are many loans underlying any given CMBS transaction, we construct an aggregate loss measure across all loans in a CMBS trust defining the expected loss for the jth CMBS trust collateralized by N loans[4] as

$$E\left[Loss_{j,t}\right] = \frac{\sum_{i=1}^{N} c_{i,t} E[Loss_{i,t}]}{\sum_{i=1}^{N} c_{i,t}} \tag{4}$$

with $c_{i,\,t}$ the outstanding principal balance on the ith loan at time t.

4. Results

We conducted daily simulations for all $N = 25,019$ CRELs underlying $J = 7$ CMBX Series 1–7 in the sample and allocated their cashflows to their seven respective CMBX trusts.[5] We aggregated the expected loss at the trust level as described in Eq. (4) for 1912 daily simulation dates from November 2007 through January 2015.

What we observe are seven time series in Fig. 1 which capture the expected losses for each CMBX series. The differentiation amongst the time series reflects the specific loan collateral attributes and actual payment status transitions which interact to inform the default intensity, λ_f. Categorically, however, the aggregated expected loss for each of the CMBX series declines as the underlying loans age, which is consistent with Eqs. (1) and (2).

Next, we capture the age of each loan on each simulation date and determine the weighted average loan age for each CMBX series in keeping with the industry practice of 'aging' loss severity data. Fig. 2 exhibits somewhat higher values than exhibited in actual cumulative losses found in the industry literature for CMBS (see, Frerich and van Heerden, 2015). Additionally, the differentiation amongst the series corresponds well with the actual loss severity experience by vintage reported by Moody's (see, Banhazl and Halpern, 2015). From our perspective, we simply note that the expected loss risk measures captured in our analysis are running lifetime estimates. Given the fact that many loans have substantial remaining time to maturity, and given considerable volatility in the CMBS market, it is possible that we may see greater convergence between our expected loss values and the actual loss experience for CRELs over time.

Anecdotal support for this perspective is found in the mapping between special serviced loan experiences for the 2007 vintage (see Barclays, 2015) with our expected losses for CMBX 5, which is made up almost exclusively with 2007 origination year loans. In Fig. 3 we see a very close mapping between the simulated expected loss and the actual special servicing

[4] We use subordination levels provided by vintage as discussed in Stanton and Wallace (2012). For further details on the loan to bond cashflow allocation procedure see Christopoulos and Jarrow (2016).

[5] CMBX 8 is excluded from this study since it was issued in January 2015.

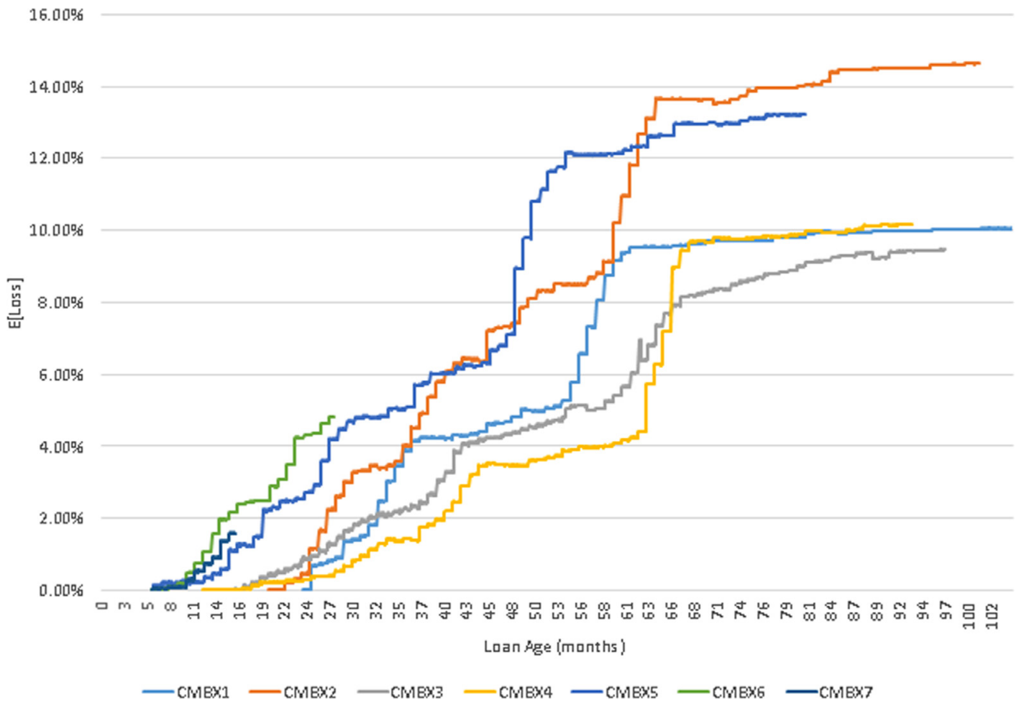

Fig. 2. Cumulative expected loss by loan age, CMBX Series 1 through 7: This chart shows the aggregated aged cumulative expected loss for CMBX Series 1 through 7.

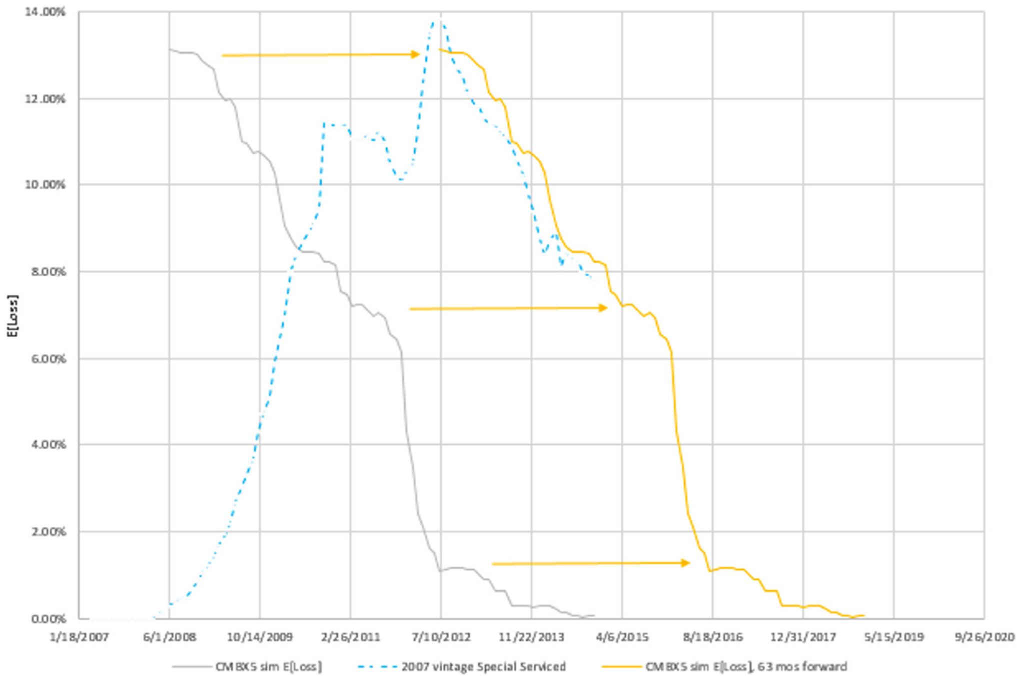

Fig. 3. Special Servicing 2007 vintage compared with CMBX Series 5: This chart shows the simulated expected loss for CMBX Series 5 (grey), the history of specially serviced loans for the 2007 vintage, and the expected losses for CMBX Series 5, 63 months forward (gold).

relationship for the 2007 vintage, 63 months forward, indicating a pleasantly coincidental 5-year forecast of losses by our model that appears to have been borne out for this vintage.[6]

[6] These results should not be surprising considering the highly similar results between the hazard rate estimates of Christopoulos and Jarrow (2016) and Christopoulos et al. (2008) and similarities in simulation approach.

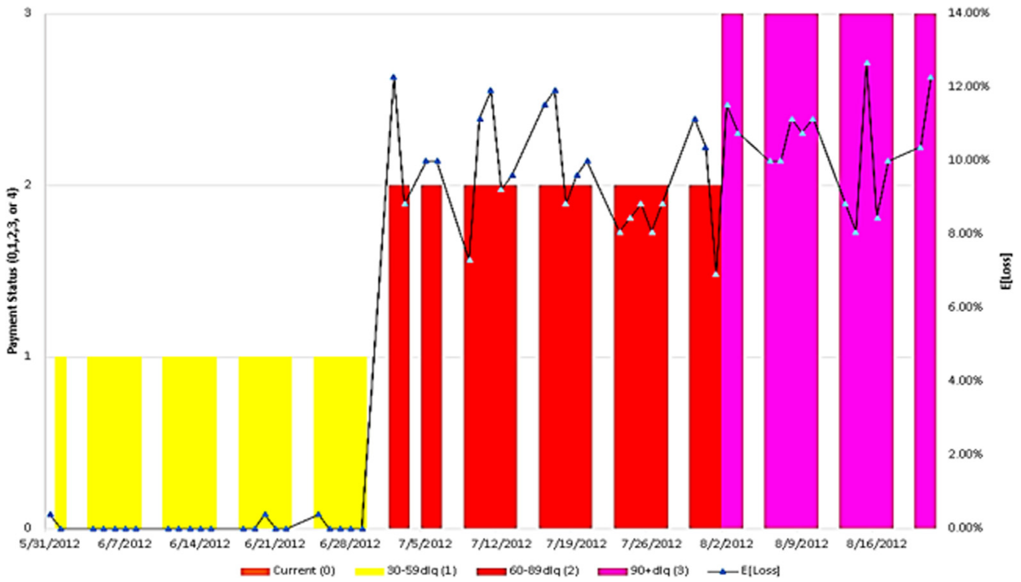

Fig. 4. Actual/historical state transition compared with simulated expected loss for one loan in CMBX Series 2: This chart shows the historical payment status for one loan compared with the simulated expected loss for the same loan.

While we establish that the simulation is performing in line with intuition as the interval $[t, T] \to 0$, this fact must be juxtaposed with the possibility of increased likelihood of default associated with the age variable in the delinquent to default transition. Age statistically has a positive and significant relationship with delinquency to default transition at the 10% level (see Christopoulos and Jarrow, 2016). This makes sense as no loans default at origination, and yet many loans may default over time up through balloon maturity as they age. Nevertheless, this finding conflicts with compression of the interval related to loan maturity.

To reconcile this, in Fig. 4 we show one example of a single loan in CMBX Series 2. As the loan's actual payment status changes in the monthly reported data, such changes inform the default intensity, λ_f, which, in turn, influences the simulated expected loss. We see in the plot, for a single loan in CMBX Series 2, the simulated expected loss (line) compared with the actual payment status (bars) over the period May 2012 through August 2012, on a daily basis. The loan's payment status in the data begins in the first two months at 30–59 days delinquent (yellow bar) as measured on the primary (left) Y-axis. In third month, the loan payment status deteriorates to 60–89 days delinquent (red bar). In the fourth month the loan payment status deteriorates further to 90+ days delinquent (magenta bar). On the secondary (right) Y-axis we measure the simulated expected loss value for this loan (line) and see that it increases in the 12 weeks of daily simulation as the loan ages even while there is a truncation of possibilities in the simulation itself as the loan moves closer to its maturity date, T.

5. Summary

This analysis provides insights into the way in which the three-part influences of i.) age (statistically), ii.) compression of the loan life interval $[t, T]$ in the simulation, and iii.) sensitivity to a changing actual information set related to the payment status of the loan, jointly interact to create a robust estimate of default and loss in the reduced form. Our approach correctly captures muted temporal variability implied by Eq. (2) while also considering particular variability at the loan level corresponding to new information and statistical influences impartially considered in the default intensity, λ_f. The compression in uncertainty in the simulation as the loan moves closer to its maturity date, is weighed against the other factors of age and change in information contained in updated data. Our analysis highlights how these seemingly conflicting aspects of loan risk are well resolved in the reduced form model of Christopoulos and Jarrow (2016) and provides further support for the use of the reduced form approach in the evaluation of CMBS default and loss risks.

Acknowledgements and funding sources

We wish to acknowledge and thank the anonymous senior portfolio manager who provided us with the private data and expert feedback that made this study possible. We also wish to thank Douglas Cumming the Editor in Chief of Finance Resarch Letters and an anonymous referee for their timely and excellent feedback on this study. We would also like to acknowledge the continued support, guidance and interest in this work by Robert Jarrow Christopoulos wishes to gratefully acknowledge the support for his work by the Department of Finance of the University of Texas-Austin, McCombs School.

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors. All funding for this project was undertaken by Christopoulos.

Appendix

In this Appendix we provide a short synopsis of key elements in Christopoulos and Jarrow (2016) pertinent to this paper.

Reduced form default model

In the case of the reduced form approach introduced by Jarrow and Turnbull (1992, 1995) and applied to CMBS by Christopoulos et al. (2008), the default trigger takes the form of default intensities. These intensities are used in the simulation in the Cox process as described in Lando (1998) to form the threshold for default in the simulation.

The reduced form model of Christopoulos and Jarrow (2016) is a credit-only model which abstracts from prepayment and liquidity risk. It models interest rate risk using a multi-factor Heath, Jarrow and Morton[7] term structure model. Credit risk is captured using a reduced-form approach first introduced by Jarrow and Turnbull (1992) and Jarrow and Turnbull (1995). The economy is simulated[8] using the Cholesky decomposition method (see Glasserman, 2003) to capture seventy-three correlated random variables in Monte Carlo simulation.

The statistically determined transition intensities articulate payment state transitions of current to delinquent (CD), delinquent to current (DC) and delinquent to default (DF), so $g \in \{(C \to D), (D \to C), (D \to F)\}$. The transition intensity process for evaluation of credit risk corresponds with the regulatory requirements of the Federal Reserve Board under CCAR.[9]

We are given a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,\mathcal{T})}, \mathbb{P})$ satisfying the usual conditions[10] with $\mathbb{P}$ the statistical probability measure. The trading interval is $[0, T]$. Traded are default free bonds of all maturities $T \in [0, T]$ with time t prices denoted $p(t, T)$, and various property indices, REITs, CRELs, CMBS bonds, and CMBX indices. The default-free spot rate of interest at time t is denoted r_t. Let $(X_t)_{t\in[0,\,T])}$ represent a vector of state variables, adapted to the filtration, describing the relevant economic state of the economy. For example, the spot rate of interest could be included in this set of state variables. We assume that markets are complete and arbitrage free so that there exists a unique equivalent martingale probability measure $\mathbb{Q}$ under which discounted prices are martingales. The discount factor at time t is $e^{-\int_0^t r_s ds}$. Because we are interested in valuing CMBS, most of the model formulation will be under the probability measure $\mathbb{Q}$.

The intensity estimates are determined from a multinomial logistic regression for hazard rate transitions utilizing 1,921,007 historical loan life transition observations from the Intex database of all loans underlying CMBX 1–8 between November 2007 and June 2015. Default is thus modelled as an intensity process. Each commercial loan i has a current, delinquent, and default intensity process that depends upon its payment status N_t, the state variable vector X_t, a vector of loan specific characteristics U^i that are deterministic (non-random), e.g. the net operating income of the underlying property at the loan origination, and time dependent loan characteristics V_t^i, e.g. the age of the loan.

The current, delinquent, and default intensity processes for each loan have the same functional form, differing only in the loan specific variables used. Default may only be arrived at from a state of delinquency, over the discrete time interval $[t, t+\Delta]$ and we then define the default intensity as

$$\lambda_f(t, U^i, X_t, V_t^i, N_t)\Delta = 1/(1 + e^{-(\varphi_f + \phi_f U^i + \psi_f X_t + \xi_f V_t^i)}), \tag{5}$$

where φ_c, ϕ_c, ψ_c, ξ_c, φ_d, ϕ_d, ψ_d, ξ_d, φ_f, ϕ_f, ψ_f, ξ_f are vectors of constants, and $\lambda_f(t, U^i, X_t, V_t^i, N_t)\Delta$ is the probability of jumping to default from delinquent.

Estimation of these intensities is under the statistical measure and given the assumption that delinquency and default risk are conditionally diversifiable, as discussed in Jarrow (2001) and Jarrow et al. (2005), these intensity functions will be equivalent under both the empirical and martingale measures. We then estimate these intensities using a multinomial logistic regression of the form

$$F_g(v) = \frac{1}{(1 + e^{-(\beta_0 + \sum_{i=1}^{m} \beta_i v_i)})} \tag{6}$$

where $F_g(v)$ represents the probability of a transition event for each of the three transition categories $g \in \{(C \to D), (D \to C), (D \to F)\}$, β is a constant vector, and v is the vector of independent variables.

Christopoulos and Jarrow (2016) validate the predictive ability of the model default probabilities using the standard techniques[11] in the credit ratings literature of: i.) the Receiver operating characteristic area under the curve (ROC AUC) and ii.) the Brier score. Briefly a ROC AUC value of 0.50 for the ROC AUC indicates a random model with no predictive ability, while a value of 1.00 indicates perfect forecasting. The Brier score is the average mean square error of a predictor with a binary

[7] See Heath et al. (1992).

[8] A technical specification of the simulation procedure used to determine these thresholds can be provided upon request.

[9] See *Appendix B* of Board of Governors of The Federal Reserve System (2015).

[10] See Protter (1990).

[11] See Medema et al. (2009), and others.

event $B = \frac{1}{N} \sum_i^N \left(\hat{p}_i - Y_i\right)^2$ where $\hat{p}_i$ is the estimated probability of an event. It holds that lower Brier scores indicate better predictive power of the model. They report a ROC AUC of 0.76 and a Brier score of 0.05 which is in keeping with the literature.[12]

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[12] See Agarwal and Bauer (2014); Güttler and Krämer (2008), and Christopoulos et al. (2008), among others.